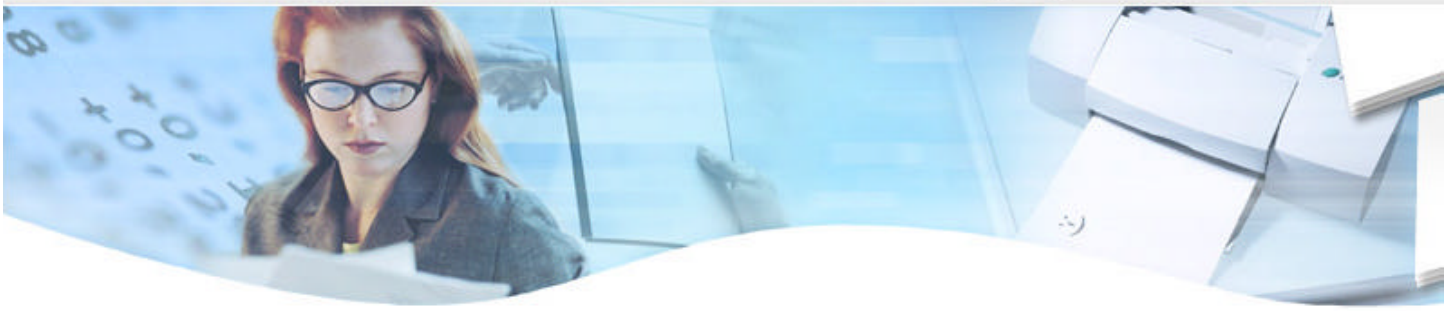


The geometry of the A4

There is a great deal of science in the size and shape of paper that we use today.

The dimensions of the paper we use in photocopiers and printers, that eventually became the norm for letters and pads, have an interesting denomination: A4, a format that is used throughout the world. A4 pages have a width of 210mm and a length of 297mm; an odd measure, since it would seem to make more sense to use round numbers. Why not 20cm by 30cm, for example?

These odd measures originated from a thoroughly considered convention, adopted by our country in 1954, still in the German system DIN (Deutsches Institut für Normung e.V.). That same convention was formalised in the ISO 216 norm of the International Organisation for Standardisation.

According to that norm there are a series of basic formats of paper that start at A0, (the largest), followed by A1, A2 , A3 and so forth down to the very small A10; 26mm by 37mm. All these formats are obtained by folding a page in half in order to obtain the format above.

This means that, for example, by folding A0 in half we obtain A1, and that by folding A4, we obtain A5. But there's much more than that in the norm. The formats were created in order to always maintain the same proportion between the sides of the paper. The sizes are rounded to the millimetre, which offers a reasonable approximation.

The rule is very practical, particularly for photocopies. By placing two A4 sheets side by side and choosing the reduction mode, it is possible for each of the original sheets be photocopied precisely to half of the resulting A4 page.

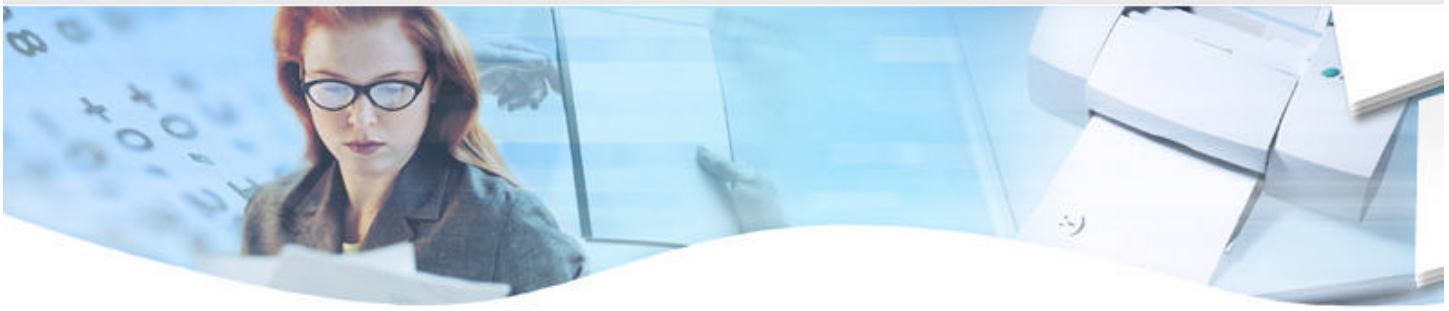
It is easy to see that not all formats would allow the same result. As an example, if the original pages were square and we wanted to photocopy two, with reduction, to the other square page, we would have to waste half of the copy page. In A4, as the proportion is maintained when we fold the page in half, there is no waste.

What format must the pages have to keep the proportions when we divide them in half? We only have to make a few simple calculations to find out. The sides of the rectangle must be in the proportion of 1 to the square root of 2, (approximately 1.4142). There is no other solution. Do the calculation for yourself and you will see that 210 x 1.4142 = 296.982; practically 297. We have found the proportions of A4.

All of this obeys perfect logic, but the starting point needs to be defined. How is A0 constructed? Curiously, this is also not arbitrary. A0 was defined with the sides in the proportion of 1 to the square root of 2, (as it would have to be), and the restriction of having an area equivalent to one square metre was added. With this, the system was perfectly defined. By chance, the A4, an ideal format for office work, resulted from this.

These norms facilitate the calculations of the weight of a ream of paper. The weight, in grams, is referred to the square metre. It is common to use, for example, 75gm of paper per square metre. This means that an A0 page of that paper weighs 75gm.

Since in an A0 page there are 16 A4 pages, we only have to make a simple calculation to verify that an A4 ream, (500 sheets), weighs 2343.75gm.



The geometry of the A4

The system defined by the ISO 216 norm has two other series of formats. Series B, used for envelopes containing documents with sheets of paper from Series A, and Series C, used for slightly smaller envelopes, that may contain fewer sheets. Therefore, as an example, if you require an envelope to send an A4 brochure, you must use a B4 envelope, (250mm x 253mm).

If you wish to send a small A4 document, you must acquire a C4 envelope, (229mm x 324mm). In this way the retailers know what they are selling.

But not even the exact dimensions of the envelopes escape mathematical logic. To define system B, the geometric average between two consecutive A formats is used. In this way, for example, to define envelope B4, we calculate the geometric average between the dimensions of A4 and A3 paper. To define C format, we use a similar process. The C4 envelope is defined by the geometric average between the A4 and B4 dimensions.

The geometric average is an average, as it would be said by La Palisse, giving intermediate dimensions between extreme values. But it is a peculiar average. It is obtained by taking the square root of the product of two values. In this way, the relative proportions are maintained. Therefore, B4 is to A4 as A3 is to B4.

This complex system has evolved throughout the centuries, becoming the major system in most parts of the world. It is thought that the first person to have the idea of normalising paper with similar rules was a German Professor of Physics, named Georg Christoph Lichtenberg (1742-1799).

In a letter that he wrote in 1786 to his friend Johann Beckmann, he discussed the aesthetic and practical advantages of using paper with all sides in proportion of 1 to the square root of 2. Although the practical advantages are clear, the aesthetic advantages are not. Today's graphic designers know that these series are not aesthetically advisable for posters or magazines.

Not even the posters that are seen on the street nor the "Expresso" magazine use the formats of the ISO 216 norm. This is one of the reasons why North Americans have not yet abandoned the "letter" format, which they have been using for many years and which measures 8 x 11 inches, (approximately 216mm x 279mm).

The practical advantages of the Lichtenberg method are, nevertheless, so evident that the French government decided to adopt it soon after the metre was instituted. In 1794, the "Loi sur le Timbre" defined several formats that correspond to the actual ISO norm. Instituted were the "grand registre", (the current A2), the "grand papier", (B3), the "moyen papier", (A3), the "petit papier", (B4), the "demi feuille", (B5), and the "effets de commerce", (1/2 B5). The only one missing was A4.

Today, the ISO norm is adopted throughout most of the world. When you pick up an A4 page, remember that you are holding a distinct piece of mathematical history.

"In" Expresso Magazine – Text by Nuno Crato